***Discrete Math1*. Handwriting Assignment #3**

**Due to 18th/April**

1. a) Is the number 0 in ∅?   
   b) Is ∅ = {∅}?  
   c) Is ∅ ∈ {∅}?
2. Let A and B be two sets. Prove that (A − B) ∩ (A ∩ B) = ∅.
3. Let A and B be two sets. Show that if A ⊆ B then A ∩ Bc = ∅.
4. Show that if n is an odd integer then ⎡n/2⎤ = (n+1)/2 .
5. Certain automobile license plates consist of a sequence of three letters followed by three digits.  
   a) If letters cannot be repeated but digits can, how many possible license plates are there?  
   b) If no letters and no digits are repeated, how many license plates are possible?
6. In how many ways can three distinct numbers be chosen from the set {1, 2, . . . , 100} such that their sum is even?
7. a) Give a recursive definition of a function named repeat that takes three parameters. The first parameter is a function from integers to integers, the second parameter is a natural number, and the third is an integer. The output of repeat(f, n, x) should be the result of applying f to x, then applying f again to the result, and so on, n times.  
   For example, suppose add1 (n) = n + 1, then   
    repeat(add1 , 3, 5) = add1 (add1 (add1 (5))) = 8  
    repeat(add1 , 0, 5) = 5  
   b) Prove that for any f ∈ Z → Z, m ∈ N, n ∈ B, x ∈ Z,  
    repeat(f, n + m, x) = repeat(f, m, repeat(f, n, x)).